EEL 711 Minor Test II Semester I 2014-2015

Answer all questions (Marks: Q.1: 24, Q.2: 16) Full Marks: 40

1. Let X_1, X_2, X_3 be i.i.d. $\mathcal{N}(1,1)$ random variables. The random vector $\underline{Y} = [Y_1\ Y_2\ Y_3]^T$ is given by

$$Y_i \triangleq \alpha X_i - \frac{X_1 + X_2 + X_3}{3} - (\alpha - 1), \quad i = 1, 2, 3,$$

where α is a deterministic real constant.

(a) Find $\underline{K}_Y$, the covariance matrix of $\underline{Y}$, in terms of α. [6]

(b) Find the range of α for which $\underline{K}_Y$ is positive definite. [4]

(c) For $\alpha = 2$, the vector $\underline{Y}$ is to be transformed to another random vector $\underline{V} = [V_1\ V_2\ V_3]^T$ such that $\underline{V} = \underline{A}\underline{Y} + \underline{b}$, $\underline{A}$ is a lower triangular matrix, and

$$\Psi_{V_1,V_2,V_3}(\jmath\omega_1, \jmath\omega_2, \jmath\omega_3) = \exp\left(\jmath[3\omega_1 + 2\omega_2 + \omega_3] - 2\left[\omega_1^2 + \omega_2^2 + \omega_3^2\right]\right).$$

Find $\underline{A}$ and $\underline{b}$. [10]

(d) For $\alpha = 1$, find the joint p.d.f. $f_{Y_1,Y_2,Y_3}(y_1, y_2, y_3)$. [4]

2. A complex-valued circular Gaussian random vector $\underline{Z} = [Z_1\ \ Z_2]^T$ has a mean vector $\underline{\mu} = [\jmath\quad -1]^T$ and a correlation matrix $\underline{R}$, whose element in the kth row and lth column is given by

$$\begin{aligned}(\underline{R})_{k,l} &= \frac{1}{2} + \jmath^k(-\jmath)^l, \quad k \neq l,\\ &= 2, \quad k = l.\end{aligned}$$

The covariance matrix of $\underline{Z}$ is $\underline{K}$. Let $\underline{X} = [X_1\ X_2]^T = \mathrm{Re}(\underline{Z})$ and $\underline{Y} = [Y_1\ Y_2]^T = \mathrm{Im}(\underline{Z})$. The vector $\underline{Z}$ is to be transformed to another random vector $\underline{W} = [W_1\ W_2]^T$ using an affine transformation such that $\underline{W} = \underline{A}\underline{Z} + \underline{b}$, and the c.f. of $\underline{W}$ is given by

$$\Psi_{W_1,W_2}(\jmath\nu_1, \jmath\nu_2) = \exp\left\{-|\nu_1|^2 - |\nu_2|^2\right\}.$$

(a) Find $\underline{A}$ and $\underline{b}$ using eigendecomposition. [8]

(b) Calculate $\mathbf{E}\left[(X_1(X_2+1)(Y_1-1)Y_2\right]$ and $\mathbf{E}\left[X_1 \mathrm{Re}(W_1)\right]$. [8]
